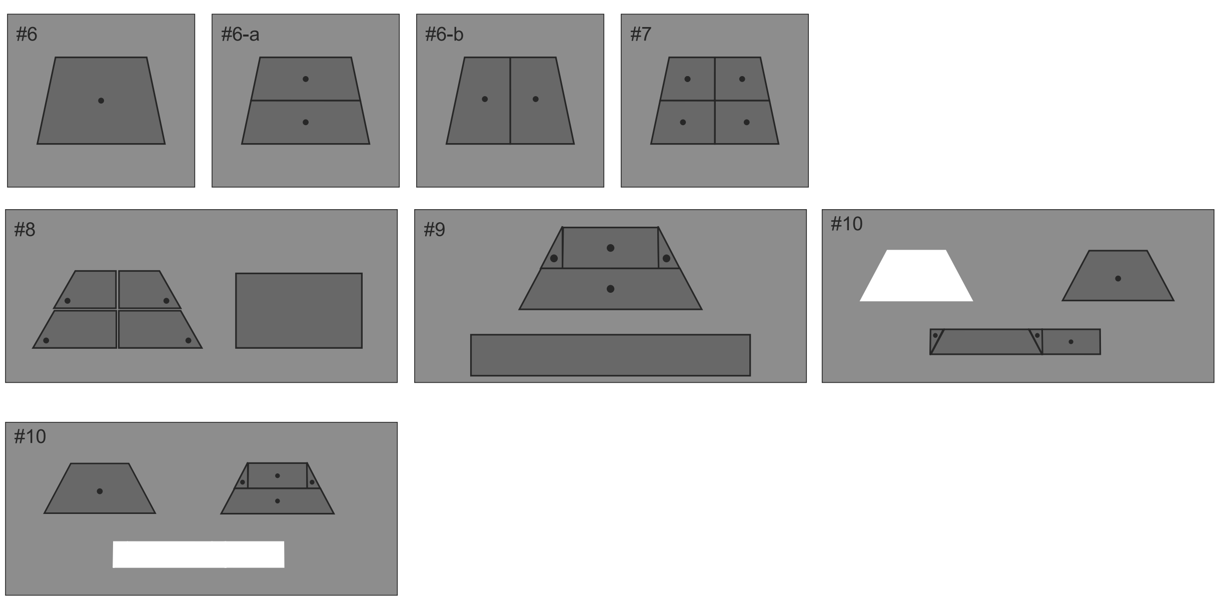
EQUIVALENCE OF THE TRAPEZOID AND RECTANGLE

Group 4, Insets #6 to #10

Materials:

- Inset #6, the whole trapezoid
- Inset #7, the trapezoid cut in four parts. The figure is cut in half from the midpoints of each base. A second cut is made at the midpoints of the oblique sides.
- Inset # 8, the large rectangle, plus the empty frame
- Inset #9, the long rectangle, plus the empty frame
- Inset #10. It includes the whole trapezoid, the trapezoid cut in four parts, and the frame for the long rectangle.



Part One: Insets #6 to #9 only.

1. Recognition of the figures in the four insets.
2. Equivalence. We want to show that the whole trapezoid is equivalent to the two rectangles.
   a. Exchange Insets #6 and #7 (the two trapezoids). They are equal.
   b. Take Inset #8 (large rectangle). Move the whole rectangle to the empty frame and back. They are equal.
   c. Try to fit the whole trapezoid in the empty frame. It will not fit. Now use the trapezoid cut into four parts. Pieces may be fit in any way. They fit in the frame.

   Therefore: The whole rectangle is equivalent to the trapezoid.

   d. Take Inset #9 frame, the second rectangle. Try first to fit the whole trapezoid. It does not fit. Try the trapezoid cut in four parts. These pieces fit.

   Therefore: The trapezoid is equivalent to the rectangle.

3. Recognition of lines between figures.

   Trapezoid: Find the major base, minor base, height, oblique sides.

   Rectangle: Find the base and height.

   Trapezoid cut into four parts: Find the base and height. The bases are divided in half. The height is divided in half.

4. Relationship of the lines in the figures.

   Why is the trapezoid equivalent to the two rectangles? Study of the lines will help us.

a. The large rectangle: The base is 1/2 the major base plus 1/2 the minor base of the trapezoid.

$$\frac{\text{Major base} + \text{Minor base}}{2} = \frac{(B + b)}{2}$$

The heights of the trapezoid and the rectangle are the same. Demonstrate with the rectangle.

Conclusion: A trapezoid is equivalent to a rectangle having as its base 1/2 the sum of the bases of the trapezoid and having the same height as the trapezoid.

Note: It is important that the pieces of the trapezoid be placed in the rectangle in a special way to determine this relationship.

b. The long rectangle: Place the pieces of the trapezoid in the frame of the rectangle.

Rectangle: The base equals two halves of the major base plus two halves of the minor base.

$$\frac{B}{2} + \frac{b}{2} + \frac{B}{2} + \frac{b}{2}$$

In the frame:

$$\frac{B}{2} + \left(\frac{b}{2} + \frac{b}{2}\right) + \frac{B}{2}$$

The height of the rectangle is 1/2 the height of the trapezoid.

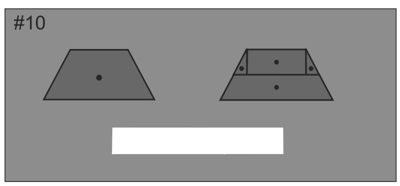
Conclusion: A trapezoid is equivalent to a rectangle having as its base the sum of the bases of the trapezoid and a height that is 1/2 the height of the trapezoid.

Note: This is the first time we have shown one figure equivalent to two others with this material.

Final Exercise: Now take two rectangles. Trace one of them on paper, for example, the long rectangle. Fold the paper in half (midpoint of the base) and form the other rectangle.

Part Two: Inset #10

This is the only trapezoid in The Advanced Montessori Method, p. 257.



1. Recognition of the figures.
2. Equivalence: The whole trapezoid will not fit in the rectangle. Exchange the two trapezoids. Fit the fractional trapezoid in the rectangle.
3. Study of lines: bases, heights, oblique sides.

The fractional trapezoid: It is divided into two smaller trapezoids. This division is at the midpoint. The upper trapezoid is again divided into three parts, a rectangle and two right triangles.

a. Find the midpoint of the height and draw a line parallel to the bases.
b. With the smaller trapezoid, draw two heights beginning at the angle of the minor base.

The rectangle: Ask the child to rebuild the rectangle. We don't need the two small triangles.

Base: The major base plus the minor base (B + b)

Height: 1/2 the height of the trapezoid (use the triangles to show this)

Conclusion: The trapezoid is equivalent to a rectangle having as its base the sum of the bases of the trapezoid and having a height _ the height of the trapezoid.

Activity: As in the previous exercises, the child draws the figures.

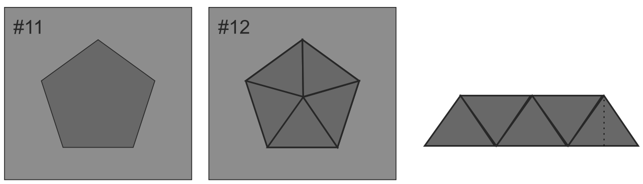
Equivalence Command Card #4

REGULAR POLYGONS

Group 5, Insets #11 to #16

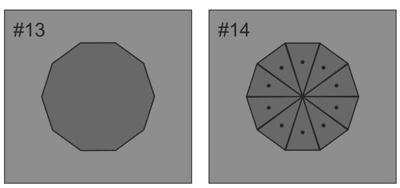
Materials:

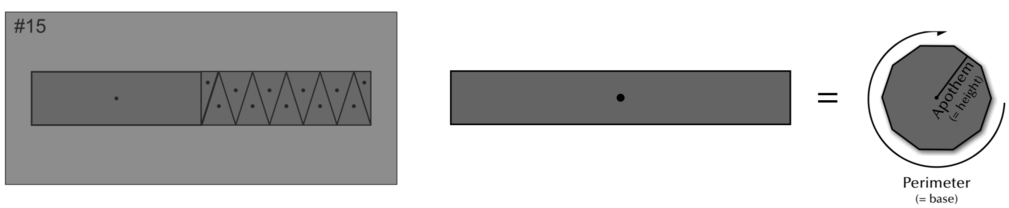
- Group 1: Insets #11 and #12, regular pentagons, one entire, one fractional



apothem = height of the trapezoid

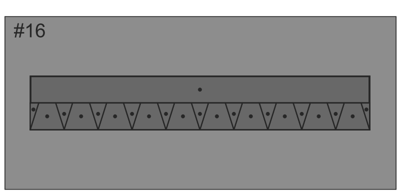
- Group 2: Insets #13 to #16, the decagon





Presentation A: The Regular Pentagon, a regular polygon with an odd number of sides.

1. Identification of the figures: pentagon. Move the inset around in its place. It is a regular pentagon.
2. Equivalence: Interchange the two insets. They are equal. There is one entire pentagon and one divided into five equal isosceles triangles.



Conclusion: A regular pentagon may be divided into as many triangles as there are sides and all of these triangles are equal.

This is all that can be done with these insets.

An apothem of a regular polygon is a line drawn from the center of the polygon perpendicular to one side.

The apothem is not the side of one of the triangles. As the number of sides of a polygon increases, the length of the apothem increases.

Activities: Equivalence Command Card #5

Presentation B:

The decagon with Insets #13 to #16, the whole decagon, the fractional decagon, the two long rectangles that are fractional.

1. Recognition of the figures: There are no empty frames in this set of insets. The frames encompass rectangles.
2. Equivalence: We want to show the equivalence of the whole decagon and the two large pieces in each rectangle frame.

The work here is similar to that already done with the trapezoids. The fractional decagon is the mediator.

a. Consider the two decagons. Interchange them. Count the pieces in each fractional decagon. There are 10 isosceles triangles (as many triangles as there are sides in the polygon).

b. Remove the triangles from the decagon inset. Try to fit in this frame the small pieces from frame #15. There are 9 isosceles triangles plus 2 half triangles. They fill the decagon frame.

3. Try to fit the triangles from the decagon in the rectangle frame. We must exchange one triangle for the two half triangles in order to fill the space.

We now know that the decagon is equivalent to this half of the rectangle.

4. Superimpose the large piece from the rectangle frame over the smaller pieces. They are equal.

Therefore: The whole decagon is equivalent to the large rectangle.

Activity: Equivalence Command Card #6.

Second Demonstration of the Decagon Insets.

This is done before passing to the Study of Lines. Exchange the small pieces from the rectangle to the decagon. Superimpose the large rectangle on the small pieces. Fill the space left in the rectangle frame with the pieces from the decagon. The rectangle frame is now filled.

Conclusion: There are two decagons in the frame of the rectangle. Count the pieces. There are 20 including the two half triangles.

Study of lines: What relationship can we find to show equivalence?

a. Take the triangular pieces from the decagon and set them in a line.

Perimeter of decagon:

b. Take half of the pieces and fit them above the others. Use the half pieces at the ends.

c. Check with the rectangle. The base of the rectangle is the same as 1/2 the perimeter of the decagon.

Now find the height.

a. Remove one triangle from the bottom of the decagon.
b. Substitute the two half triangles in this space, then remove one of them. Replace it in the decagon inset. We now have a line from the center of the decagon to the midpoint of one of its sides - the apothem.
c. Take the rectangle and fit it in the space of the decagon. The height of the rectangle is the same as the apothem of the decagon.

Conclusion: A regular polygon is equivalent to a rectangle which has as its base 1/2 the perimeter (semi-perimeter) of the decagon, and a height that is equal to the apothem of the decagon.

Area of a polygon = 1/2 perimeter x apothem